Binary Number Conversions

Professor Don Colton

Brigham Young University Hawaii

Several number bases are common in the Information Technology field, including networking. Binary (base two) is the underlying model for most data communications. Octal (base eight) and hexadecimal (base 16, also called "hex") are popular shorthands for presenting binary data. Decimal (base ten) is the number base we most commonly use. It is useful and occasionally important to be able to convert numbers from one base to another.

1 Binary Shortcut

How do we know that 2^{16} is about 64 thousand? To quickly answer such questions it is helpful to know two facts: (a) the powers of two up to 10, and (b) the fact that 2^{10}=1024 is almost one thousand.

Exact Powers of Two				Approximate	
$2^1 =$	2	$2^6 =$	64	$2^{10} =$	thousand
$2^2 =$	4	$2^7 =$	128	$2^{20} =$	million
$2^3 =$	8	$2^8 =$	256	$2^{30} =$	billion
$2^4 =$	16	$2^9 =$	512	$2^{40} =$	trillion
$2^5 =$	32	$2^{10} =$	1024	$2^{50} =$	quadrillion

In the case of 2^{16} we know that 2^6 is 64, so $2^{16}=2^6 \times 2^{10}$ is about 64 thousand and $2^{26}=2^6 \times 2^{10} \times 2^{10}$ is about 64 million.

Similarly, 2^4 is 16, so 2^{14} is about 16 thousand and 2^{24} is about 16 million.

2 32 Bits

On the Internet, things are organized by bits. Take the following string of bits.

1011 1010 0100 1110 0101 0010 0000 0111

Generally we write IPv4 addresses in base 10. The first eight bits (1011 1010) are called the first octet. They translate to 186. The second eight bits (0100 1110) are called the second octet. They translate to 78. The third eight bits (0101 0010) are called the third octet. They translate to 82. The fourth eight bits (0000 0111) are called the fourth octet. They translate to 7. Generally we would write this address in "dotted-quad" notation, with each octet in base 10, like this.

186.78.82.7

3 Binary Numbers

The binary number system uses only two digits: zero and one. Often instead of calling them digits, they are called bits, which is short for bi(nary digi)ts. With this restriction, instead of counting 1, 2, 3, 4, 5, ... we count 1, 10, 11, 100, 101, ... It is a fully-functional numbering system, able to represent even numbers like π (3.1415) as 11.001 (with a bunch more digits). It takes about three times as many bits to express a number in binary as it does in base 10.

Data transmission on the Internet is generally thought of in terms of binary numbers because the underlying physical phenomena can be comfortably described in that way. Because the underlying nature is generally dealt with in binary, it is important for networking practitioners to also work in binary when appropriate.

3.1 Divide and Shift

Divide and Shift is one easy method to convert a number from base 10 to binary. Divide it by two repeatedly. Each time you divide, take the remainder and add it to the front of the binary number you are constructing. Here for example we will convert 2005 (the year this tutorial was written).

Taking 2005, we divide by two to get 1002.5. But let's keep it to whole numbers. Dividing we get 1002 with a remainder of 1. We copy the remainder to the front of the binary number we are building.

2005		our starting number
1002	1	divide and copy
501	01	divide and copy
250	101	continue
125	0101	continue
62	1 0101	continue
31	01 0101	continue
15	101 0101	continue
7	1101 0101	continue
3	1 1101 0101	continue
1	11 1101 0101	continue
0	111 1101 0101	continue
0	0111 1101 0101	okay, stop

We could keep putting zeroes at the front, but there is no point.

2005 in base 10 becomes 111,1101,0101 in base 2.

To convert a number from base 2 to base 10, do the same thing in reverse. Double the base 10 number and add the first bit of the base 2 number. We will covert our second octet from above.

0	0100 1110	our starting number
0	100 1110	double and add zero
1	00 1110	double and add one
2	0 1110	double and add zero
4	1110	double and add zero
9	110	double and add one
19	10	double and add one
39	0	double and add one
78		double and add zero

0100,1110 in base 2 becomes 78 in base 10.

3.2 Powers of Two

Powers of Two is another easy method to convert a number from base 10 to binary. We start with a table of powers of two. We subtract the largest power repeatedly. Then we add up the results. Again we will convert 2005.

1	1
2	10
4	100
8	1000
16	1 0000
32	10 0000
64	100 0000
128	1000 0000
256	1 0000 0000
512	10 0000 0000
1024	100 0000 0000
2048	1000 0000 0000

2005		our starting number
1024	100 0000 0000	largest power of two
981		after we subtract
512	10 0000 0000	largest power of two
469		after we subtract
256	1 0000 0000	largest power of two
213		after we subtract
128	1000 0000	largest power of two
85		after we subtract
64	100 0000	largest power of two
21		after we subtract
16	1 0000	largest power of two
5		after we subtract
4	100	largest power of two
1		after we subtract
1	1	largest power of two
0		done

1024	100 0000 0000
512	10 0000 0000
256	1 0000 0000
128	1000 0000
64	100 0000
16	1 0000
4	100
1	1
2005	111 1101 0101

Similarly we can add up the powers of two to get the answer in base 10.

78	0100 1110
2	10
4	100
8	1000
64	100 0000

$64 + 8 + 4 + 2 = 78$, our answer.

Students should be able to convert numbers using either method, and should be confident with at least one method.

3.3 Numbers to Memorize

To speed up calculations, it is helpful to memorize the sequence of powers of two, at least through 1024. When I say memorize the sequence, I don't mean that you remember that 64 is the sixth number, but rather that 64 comes after 32 and before 128. When we learn the alphabet, we often do not know that T is the 20th letter, but we know it is part of the sequence "Q R S T U V W."

It is also helpful to know the powers of two subtracted from 256, which are also the negative powers of two; a series of all ones followed by a series of all zeroes.

Negative Powers of Two

255	1111 1111	256-1
254	1111 1110	256-2
252	1111 1100	256-4
248	1111 1000	256-8
240	1111 0000	256-16
224	1110 0000	256-32
192	1100 0000	256-64
128	1000 0000	256-128

There are four numbers that are particularly important. They are 0*, 0*1, 1*, and 1*0, with any number of bits.

0*	... 0000 0000 0000	all zeroes
0*1	... 0000 0000 0001	zeroes and a one
1*	... 1111 1111 1111	all ones
1*0	... 1111 1111 1110	ones and a zero

Powers of Two

1	1
2	10
4	100
8	1000
16	1 0000
32	10 0000
64	100 0000
128	1000 0000
256	1 0000 0000
512	10 0000 0000
1024	100 0000 0000

Negative Powers of Two

255	1111 1111	256-1
254	1111 1110	256-2
252	1111 1100	256-4
248	1111 1000	256-8
240	1111 0000	256-16
224	1110 0000	256-32
192	1100 0000	256-64
128	1000 0000	256-128

Four Important Numbers

There are four numbers that are particularly important. They are 0*, 0*1, 1*, and 1*0, with any number of bits.

0*	... 0000 0000 0000	all zeroes
0*1	... 0000 0000 0001	zeroes and a one
1*	... 1111 1111 1111	all ones
1*0	... 1111 1111 1110	ones and a zero