CS 252 – Introduction to Computational Theory
Final Exam – Winter 2002

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Instructions: Points may be deducted (up to 10%) for failure to follow instructions. The rules are designed to make grading as easy as possible, and as fair as possible. Some of the rules are common sense, but I state them here to be explicit.

Grading: Final is 320 points, scaled to be 30% of the final grade. The other components are: midterm 10%, attendance 20%, lab 10% extra credit, reading 20%, homework 20%, for a total of 110% including extra credit.

Paper: I provide exam paper and scratch paper. I will give you some to start with, and you are welcome to take as much more as you think you need. After the exam, you may keep your scratch paper or throw it away. I don't want it. After the test, you may keep these questions. Turn in your answers only.

Identity: DO NOT WRITE YOUR NAME ANYWHERE ON THE ANSWER PAGES YOU TURN IN. I want to grade it without knowing whose test it is. This is to help guarantee fair grading for everyone. Instead, you will be given a random business card with a unique Test ID number written on the back of it. Write your name on the back of the business card, and write your Test ID (but not your name) in the upper RIGHT ($\nearrow$) corner on all pages you submit. After the tests are graded, I will match the names to the tests. Graded tests will be left outside my office door for you to pick up.

Format: Write on the front of each page. Do not write on the back; leave it blank. Leave a blank margin around each sheet, about the same size as the margins on this sheet, maybe bigger. Make sure your answer pages are clearly labeled. You may combine answers on the same page if you wish. Make sure your answers are neatly labeled and separated from each other, and presented in order (1, 2, 3, not 1, 3, 2, etc.). Help my eyes find your answer immediately. Do not staple, fold, or tear your pages. I will staple them myself, as needed.

Discuss: Some questions ask you to discuss a word or phrase. This means write briefly the most important things about that topic. For instance, what it is, how is it done, how it is used, what issues are involved, etc.

Let $L_1 = \{w | w \in a^i b^j c^k, i + k \leq j\}$

Let $L_2 = \{w | w \in \{a, b\}^*, w$ has more a's than b's $\}$

Let $L(G)$ be the language generated by G, the following grammar:

$$\begin{array}{lcl} S & \longrightarrow & X \mid Y \mid a \mid b \mid \Lambda \\ X & \longrightarrow & aXa \mid bYb \\ Y & \longrightarrow & bXb \mid aYa \end{array}$$

Let L_3 be the language generated by the regular expression: $(a + b)^*abb$

Let $L_4 = \{w | w \in a^i b^i c^i, i > 1\}$.

Part I: Regular Languages

Problem 1: (20 points) Convert the regular expression $(a+b)^*baa$ into an NFA.

Problem 2: (20 points) Convert the regular expression $ab(a+b)^*b$ into a DFA.

Problem 3: (10 points) Discuss the pumping lemma for finite automata.

Problem 4: (10 points) Discuss "decidable."

Problem 5: (10 points) Discuss "closure."

Part II: Context Free Languages

Problem 6: (25 points) Construct a Context Free Grammar that generates L_1.

Problem 7: (10 points) Give a derivation tree that enables you to derive $a^2b^5c^1$ using your grammar.

Problem 8: (10 points) Do you think your grammar is ambiguous? Why?

Problem 9: (25 points) Construct a Context Free Grammar in Chomsky Normal Form that generates L_2.

Problem 10: (20 points) Is L_2 a regular language? Prove your answer.

Problem 11: (25 points) Describe $L(G)$.

Problem 12: (25 points) Write a grammar in Chomsky Normal Form that generates L_3.

Problem 13: (10 points) Using your grammar, give a derivation of *abbabb*.

Problem 14: (10 points) Discuss the pumping lemma for context free grammars.

Problem 15: (10 points) Must a PDA be non-deterministic? Why?

Part III: Turing Machines

Problem 16: (40 points) Create a Turing Machine to accept L_4.

Problem 17: (10 points) Discuss "post machines" (chapter 20).

Problem 18: (10 points) Discuss "Universal Turing Machine" (p.552).

Problem 19: (10 points) Discuss "the Chomsky Hierarchy of Grammars" (p.573).

Problem 20: (10 points) Discuss "recursively enumerable" (chapter 24).